**MATHEMATICS SPECIALIST**

**MAWA Year 12 Examination 2018**

**Calculator-free**

# Marking Key

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The release date for this exam and marking scheme is

* **the end of week 1 of term 4, 2018**

**Question 1 (5 marks)**

|  |
| --- |
| Solution |
|   Hence  Also the argument of  lies in the fourth quadrant with Thus  for integer  so  Hence the four solutions are  where  and  .Restricting to the given range requires that  where  and .  |
| Mathematical behaviours | Marks |
| * states the correct value for
* gives the correct value for
* calculates four distinct solutions of the equation (one mark for 2 or 3)
* restricts the arguments to the appropriate range
 | 1121 |

**Question 2 (a) (4 marks)**

|  |
| --- |
| Solution |
|  Since it follows that   |
| Mathematical behaviours | Marks |
| * identifies correct double angle formula to use
* simplifies the integral to requiring the anti-derivative of
* integrates correctly
* evaluates the indefinite integral at the end points
 | 1111 |

**Question 2(b) (4 marks)**

|  |
| --- |
|  Solution |
|  If we put  then we find that      |
| Mathematical behaviours | Marks |
| * calculates  correctly
* substitutes into integral changing the limits appropriately
* integrates the expression correctly
* substitutes the boundary values and simplifies to a suitable form
 | 1111 |

**Question 2 (c) (4 marks)**

|  |
| --- |
| Solution |
|  If we put  then  and the integral  Hence if the integral equals  we conclude that    |
| Mathematical behaviours | Marks |
| * identifies that
* identifies the most appropriate substitution
* evaluates the integral correctly and thereby
* deduces the correct value of
 | 1111 |

**Question 3 (a) (2 marks)**

|  |
| --- |
| Solution |
|   |
| Mathematical behaviours | Marks |
| * matches one graph correctly
* matches a second graph correctly
 | 11 |

**Question 3 (b)(i) (3 marks)**

|  |
| --- |
| Solution |
|        |
| Mathematical behaviours | Marks |
| * uses implicit differentiation to determine
* calculates  and  at
* correct conclusion
 | 111 |

**Question 3 (b)(ii) (4 marks)**

|  |
| --- |
| Solution |
|     |
| Mathematical behaviours | Marks |
| * separates the variables
* determines the correct anti-derivaties
* calculates the constant correctly
* states the required particular solution
 | 1111 |

**Question 4 (a) (3 marks)**

|  |
| --- |
| Solution |
|  |
| Mathematical behaviours | Marks |
| * draws both vertical asymptotes
* draws correct graph for
* draws correct graph for
 | 111 |

**Question 4 (b) (3 marks)**

|  |
| --- |
| Solution |
|  |
| Mathematical behaviours | Marks |
| * draws vertical asymptotes at  and at
* draws correct graph for
* draws correct graph for  and for
 | 111 |

**Question 5 (a) (2 marks)**

|  |
| --- |
| Solution |
|  The equation $x^{2}+y^{2}+z^{2}-8x+12y-24z+171=0$ can be rewritten as$(x-4)^{2}+(y+6)^{2}+(z-12)^{2}=4^{2}+6^{2}+12^{2}-171=25$ (\*)So the centre C has coordinates $\left(4,-6,12\right)$ and the radius is $\sqrt{25}=5$ |
| Mathematical behaviours | Marks |
| * obtains co-ordinates of C
* calculates radius correctly
 | 11 |

**Question 5 (b) (3 marks)**

|  |
| --- |
|  Solution |
| The point Alieson the line segment $\vec{OC} $ and on the sphere S.So Ahas coordinates $\left(4t,-6t,12t\right)$ for some $t$Substituting into the equation for S gives$(4t-4)^{2}+(-6t+6)^{2}+(12t-12)^{2}=25$  i.e. $196(t-1)^{2}=25.$ i.e. $t-1=\pm 5/14$$t=9/14$ gives the point closest to O**,** so the coordinates of Aare  .Alternative method:Distance of the centre of the sphere from the origin is  Radius of sphere is  so required point is  along the line joining O to  Hence the point A is as before |
| Mathematical behaviours | Marks |
| * obtains the correct form of the co-ordinates A in terms of a parameter
* solves for the parameter
* derives the appropriate co-ordinates of A

ALTERNATIVE* determines distance of centre from origin
* determines required point is 9/14ths along the line OC
* derives the appropriate co-ordinates of A
 | 111 1 1 1 |

**Question 5 (c) (3 marks)**

|  |
| --- |
| Solution |
|  The vector $\vec{OA}=2i-3j+6k$is normal to P.So $2x-3y+6z=c$ (\*) is a Cartesian equation of P. Since A  lies on P,  So $2x-3y+6z=63 $is a Cartesian equation of P.  |
| Mathematical behaviours | Marks |
| * recognises the normal to the plane
* writes down the correct form of the equation of the plane (\*)
* evaluates the constant correctly
 | 111 |

**Question 6 (a) (2 marks)**

|  |
| --- |
| Solution |
|  FalseThe confidence interval may contain NONE of the original population. For example, if the population consists just of 0’s and 1’s, and the sample size is large enough, then $$0<\overbar{X}-E<\overbar{X}+E<1.$$ |
| Mathematical behaviours | Marks |
| * states correct answer
* gives a valid reason
 | 11 |

**Question 6 (b) (2 marks)**

|  |
| --- |
|  Solution |
| TrueThe probability that any one confidence interval will contain the mean is equal to the confidence level, i.e. 90% or 0.9 |
| Mathematical behaviours | Marks |
| * states correct answer
* gives a valid reason
 | 11 |

**Question 6 (c) (2 marks)**

|  |
| --- |
| Solution |
| False Because the samples are independent and random it is possible that NONE of the confidence intervals will contain $μ$  |
| Mathematical behaviours | Marks |
| * states correct answer
* gives a valid reason
 | 11 |

**Question 6 (d) (3 marks)**

|  |
| --- |
| Solution |
| TrueThe probability that exactly 9 of the 10 confidence intervals will contain $μ $is $B\left(10,9,0.9\right)=\left(\begin{matrix}10\\9\end{matrix}\right)×0.9^{9}×0.1^{1}=10×0.9^{9}×0.1=0.9^{9} $(\*) On the other hand, the probability that all of the 10 confidence intervals will contain $μ $is $B\left(10,10,0.9\right)=0.9^{10}.$ (\*\*)Clearly $0.9^{9}>0.9^{10}.$   |
| Mathematical behaviours | Marks |
| * states correct answer
* derives the correct expressions (\*) and (\*\*) for the respective probabilities
 | 11+1 |

**Question 7 (5 marks)**

|  |
| --- |
| Solution |
|   |
| Mathematical behaviours | Marks |
| * shows circle with correct centre and radius
* shows correct wedge with the correct angles
* shades required area indicating that boundaries should be included
 | 1+111+1 |

