**MATHEMATICS SPECIALIST**

**MAWA Year 12 Examination 2018**

**Calculator-free**

# Marking Key

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The release date for this exam and marking scheme is

* **the end of week 1 of term 4, 2018**

**Question 1 (5 marks)**

|  |  |
| --- | --- |
| Solution | |
| Hence  Also the argument of  lies in the fourth quadrant with  Thus  for integer  so  Hence the four solutions are  where  and  .  Restricting to the given range requires that  where  and . | |
| Mathematical behaviours | Marks |
| * states the correct value for * gives the correct value for * calculates four distinct solutions of the equation (one mark for 2 or 3) * restricts the arguments to the appropriate range | 1  1  2  1 |

**Question 2 (a) (4 marks)**

|  |  |
| --- | --- |
| Solution | |
| Since      it follows that | |
| Mathematical behaviours | Marks |
| * identifies correct double angle formula to use * simplifies the integral to requiring the anti-derivative of * integrates correctly * evaluates the indefinite integral at the end points | 1  1  1  1 |

**Question 2(b) (4 marks)**

|  |  |
| --- | --- |
| Solution | |
| If we put    then we find that | |
| Mathematical behaviours | Marks |
| * calculates  correctly * substitutes into integral changing the limits appropriately * integrates the expression correctly * substitutes the boundary values and simplifies to a suitable form | 1  1  1  1 |

**Question 2 (c) (4 marks)**

|  |  |
| --- | --- |
| Solution | |
| If we put  then  and the integral    Hence if the integral equals  we conclude that | |
| Mathematical behaviours | Marks |
| * identifies that * identifies the most appropriate substitution * evaluates the integral correctly and thereby * deduces the correct value of | 1  1  1  1 |

**Question 3 (a) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * matches one graph correctly * matches a second graph correctly | 1  1 |

**Question 3 (b)(i) (3 marks)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * uses implicit differentiation to determine * calculates  and  at * correct conclusion | 1  1  1 |

**Question 3 (b)(ii) (4 marks)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * separates the variables * determines the correct anti-derivaties * calculates the constant correctly * states the required particular solution | 1  1  1  1 |

**Question 4 (a) (3 marks)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * draws both vertical asymptotes * draws correct graph for * draws correct graph for | 1  1  1 |

**Question 4 (b) (3 marks)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * draws vertical asymptotes at  and at * draws correct graph for * draws correct graph for  and for | 1  1  1 |

**Question 5 (a) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
| The equation can be rewritten as  (\*)  So the centre C has coordinates and the radius is | |
| Mathematical behaviours | Marks |
| * obtains co-ordinates of C * calculates radius correctly | 1  1 |

**Question 5 (b) (3 marks)**

|  |  |
| --- | --- |
| Solution | |
| The point Alieson the line segment and on the sphere S.  So Ahas coordinates for some  Substituting into the equation for S gives      i.e. i.e.  gives the point closest to O**,** so the coordinates of Aare  .  Alternative method:  Distance of the centre of the sphere from the origin is  Radius of sphere is  so required point is  along the line joining O to  Hence the point A is as before | |
| Mathematical behaviours | Marks |
| * obtains the correct form of the co-ordinates A in terms of a parameter * solves for the parameter * derives the appropriate co-ordinates of A   ALTERNATIVE   * determines distance of centre from origin * determines required point is 9/14ths along the line OC * derives the appropriate co-ordinates of A | 1  1  1  1  1  1 |

**Question 5 (c) (3 marks)**

|  |  |
| --- | --- |
| Solution | |
| The vector is normal to P.  So (\*) is a Cartesian equation of P.  Since A  lies on P,  So is a Cartesian equation of P. | |
| Mathematical behaviours | Marks |
| * recognises the normal to the plane * writes down the correct form of the equation of the plane (\*) * evaluates the constant correctly | 1  1  1 |

**Question 6 (a) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
| False  The confidence interval may contain NONE of the original population.  For example, if the population consists just of 0’s and 1’s, and the sample size is large enough, then | |
| Mathematical behaviours | Marks |
| * states correct answer * gives a valid reason | 1  1 |

**Question 6 (b) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
| True  The probability that any one confidence interval will contain the mean is equal to the confidence level, i.e. 90% or 0.9 | |
| Mathematical behaviours | Marks |
| * states correct answer * gives a valid reason | 1  1 |

**Question 6 (c) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
| False  Because the samples are independent and random it is possible that NONE of the confidence intervals will contain | |
| Mathematical behaviours | Marks |
| * states correct answer * gives a valid reason | 1  1 |

**Question 6 (d) (3 marks)**

|  |  |
| --- | --- |
| Solution | |
| True  The probability that exactly 9 of the 10 confidence intervals will contain is  (\*)  On the other hand, the probability that all of the 10 confidence intervals will contain is (\*\*)  Clearly | |
| Mathematical behaviours | Marks |
| * states correct answer * derives the correct expressions (\*) and (\*\*) for the respective probabilities | 1  1+1 |

**Question 7 (5 marks)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * shows circle with correct centre and radius * shows correct wedge with the correct angles * shades required area indicating that boundaries should be included | 1+1  1  1+1 |

